

Correlation Properties of the Electron-Hole Plasma Interacting with the Exciton Gas and the Formation of Inhomogeneous State.

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(Dated:)

Abstract

The correlation properties of the cold system consisting of the electron-hole plasma interacting with the exciton gas are analyzed. It is shown that the homogeneous state of the system is unstable and in the stationary state the densities of the electron-hole plasma and exciton gas are modulated.

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The problem of the existence of the Bose condensed state of the exciton gas in semiconductors is the subject of the experimental and theoretical investigations for a long time. However, there is no undoubted proof of the existence of this state until now.

Recently, the properties of the cold exciton gas are investigated intensively, both experimentally and theoretically, in double 2D quantum wells, besides, electrons are localized on one plane of these wells and the holes are localized on the other plane. The state of the cold exciton system being observed in the experiments of different experimental groups [1], [2], [3], [5], [8], [10], [11], [4], [6], [7] is nonhomogeneous, and the density of the system in these experiments is modulated in space.

The theory of this phenomenon based on the Turing kinetic mechanism of the instability [12] have been proposed in works [8], [9]. In this theory the system of nonlinear diffusion equations with sources of particles for electrons, holes and excitons are proposed and solved. The solution of these equations demonstrates the state with the periodically modulated density of the exciton system only if the constant describing the decay of excitons depends on their density. However, in the low density approximation this dependence should be neglected. Moreover, this mechanism has the classical character and does not describe the systems with the quantum coherency, but for the sufficiently small temperatures one can suppose the existence of the coherency in the exciton system [15], [16], [18].

Another theory, based on the supposition of the existence of the attraction between excitons at large distances and the formation of the liquid phase in the low-density exciton system due to this attraction, has been proposed in the work [13]. The supposition of the existence of the liquid phase in the low-density exciton system is baseless. The existence of the attractive part of the interparticle interaction potential in 2D Bose systems results in the collapse of the system to the densities of the order of the squared inverse radius of the repulsive core [?]. However, for these densities excitons can not exist as individual particles, they are destroyed and form the electron-hole liquid. Note that in the usual 3D case, as it was shown in the work [15], the scattering amplitude for the isotropic excitons is positive and is equal to the value $a = \frac{13\pi}{3}a_B$, where a_B is the effective Bohr radius, and just the scattering amplitude, but not the bare potential, determines the properties of the system in the case of small densities. The effective Bohr radius has the form $a_B = \frac{\hbar^2}{m^*(e^*)^2}$; where \hbar is the Planck constant; $m^* = \frac{m_e m_h}{m_e + m_h}$ is the reduced mass, $m_{e,h}$ are electron (e) and hole (h) masses, which are supposed to be isotropic and of the same order $m_e \sim m_h$; e^* is the effective charge in the semiconductor $e^* = \left(\frac{e^2}{\epsilon_0}\right)^{1/2}$; where e is the bare electron charge and ϵ_0 is the static dielectric constant of the semiconductor. As the result, the 3D exciton system of small density can exist only in the gas phase. The repulsion for the interaction between excitons in 2D system, where electrons and holes are localized on different planes, is sharply defined than in the usual 3D case due to the geometrical constraint. In this case the formation of the exciton liquid phase is possible only for the densities $n_{ex} \geq 1/a_{ex}^2$, where a_{ex} is the size of the exciton. The size of the exciton is equal to the effective Bohr radius $a_{ex} = a_B$ in the case of the small distance l between the planes of the quantum wells $l \ll a_B$, and $a_{ex} \gg a_B$ for the large distance between planes when $l \gg a_B$. If the exciton density is sufficiently large $n_{ex} \geq 1/a_{ex}^2$ the excitons can not be considered as the individual particles. In this case the electron-hole system represents the strongly interacting electron-hole liquid.

In the present work the Coulomb correlations in the cold electron-hole plasma as well as the quantum effects and the coherency of the exciton subsystem in quantum wells are taken into account. We suppose that the external source is stationary, has the frequency larger than the semiconductor gap Δ_g and creates electrons and holes being localized on different planes. Moreover, the region of the action of the electromagnetic field is supposed to be sufficiently small. In this case the electron-hole plasma being created by the external source propagates along 2D planes, besides, the electrons propagate along one plane and the holes along the other plane. During this propagation the electron-hole plasma loses the energy giving away the energy to the phonon system and comes to the equilibrium state with the small temperature T . Note that the parameters of the stationary state, such that the densities of the electron-hole plasma n and the exciton gas n_{ex} , are defined by the equilibrium between incoming rate of the electrons, holes and excitons and outgoing rate of these particles.

Thus, the model being analyzed represents the quasi-equilibrium electron-hole plasma from which the exciton gas is created as the result of the formation of the electron-hole bound states. In the present work we consider the properties of the stationary state of this system, namely, the properties of the electron-hole plasma of the small density $na_B^3 \ll 1$ interacting with the exciton gas of small density $n_{ex}a_B^3 \ll 1$.

The small density of the electron-hole plasma and the exciton gas are necessary if we suppose the existence both electron-hole plasma and the exciton gas. In the opposite case of the large density $na_B^3 \gg 1$ the excitons are destroyed and can not exist as the individual particles. This case is not under consideration in this work. Note that the experiments [1], [5], [8] demonstrate the existence of the excitons, and this means the existence of small density of the electron-hole plasma in these experiments.

We analyze the correlation properties of the electron-hole plasma of the small density and the interaction of this

plasma with the exciton gas. The energy of the ground state of the electron-hole plasma has a minimum value as the function of the density at some density n_0 such that $n_0 a_B^2 \sim 1$. As a result, the electron-hole plasma tends to form the liquid electron-hole drops and the homogeneous state of the system becomes unstable. The inhomogeneous state of the electron-hole plasma creates the periodic mean field which acts on the exciton gas and makes the density of the exciton system modulated in space.

We suppose that the system consisting of the electron-hole plasma interacting with the exciton gas is in the stationary state with some small temperature T obeying the inequality $T \ll \varepsilon_F^{(e,h)} \ll E_b^{(ex)}$; where $\varepsilon_F^{(e,h)}$ are the Fermi energies of the electrons and the holes correspondingly $\varepsilon_F^{(e,h)} = \frac{p_F^2}{2m_{e,h}}$; the Fermi momentum p_F in 2D case is expressed by the equality $p_F = (2\pi n)^{1/2}$; the average density n of the electron-hole plasma is equal to the density of each component $n = n_e = n_h$; $E_b^{(ex)}$ is the exciton bound energy $E_b^{(ex)} = \frac{\hbar^2}{2m^* a_B^2}$. Below we use the system of units in which $\hbar = m^* = e^* = 1$.

EFFECTIVE ACTION

To analyze the properties of the exciton system interacting with the electron-hole plasma the effective action for the slow fields is derived. In this paper the properties of the stationary state of this system is under consideration, but in future the kinetics of the formation of this state is supposed to be analyzed. For this reason the Keldysh-Schwinger technique for the nonequilibrium processes [17], [19] in the functional integral formulation (see [20], for example) is used for the derivation of the effective action of the slow fields. The generation functional for the considering system can be written as

$$Z = \int \prod_{\alpha} D\psi_{\alpha} D\bar{\psi}_{\alpha} \exp \left(i S_{e-h} + i \oint dt d^2 r (\bar{J}_{\alpha} \psi_{\alpha} + \bar{\psi}_{\alpha} J_{\alpha}) \right) \quad (1)$$

The action of the electron-hole plasma in the electron-hole representation has the form

$$S_{eh} = \oint d^3 x d^3 x' \left\{ \sum_{\alpha=e,h} \bar{\psi}_{\alpha}(x) \left[i\partial_t + \mu_{\alpha} - \varepsilon_{\alpha}(\widehat{\vec{p}}) \right] \psi_{\alpha}(x) \delta(x - x') - \right. \\ \left. - \frac{1}{2} \sum_{\alpha,\beta} (-1)^{\alpha+\beta} \bar{\psi}_{\alpha}(x) \psi_{\alpha}(x) \widehat{U}_C \bar{\psi}_{\beta}(x') \psi_{\beta}(x') \delta(t - t') \right\} \quad (2)$$

here $x = (t, \vec{r})$ denotes the collection of time t and space coordinates \vec{r} in 2D; ψ_e and ψ_h are fermion (Grassmann) fields of electrons and holes; $\varepsilon_e(\widehat{\vec{p}})$ and $\varepsilon_h(\widehat{\vec{p}})$ are the dispersive laws of the electron and hole bands $\varepsilon_{\alpha}(\widehat{\vec{p}}) = \frac{1}{2} \Delta_g + \frac{1}{2m_{\alpha}} \widehat{\vec{p}}^2$, where Δ_g is the semiconductor gap; $\widehat{\vec{p}} = -i\vec{\nabla}$ is the momentum operator; $\widehat{U}_C = U_C(\vec{r} - \vec{r}') = \frac{e^2}{\varepsilon_0 |\vec{r} - \vec{r}'|} = \frac{1}{|\vec{r} - \vec{r}'|}$ is the Coulomb interaction. In the Keldysh-Schwinger technique the time variable t changes along the double time contour with return.

The effective action for the electron-hole plasma with the formation of the exciton system have been obtained in the functional-integral technique in [23], [24]. In this paper we give more simple derivation of the effective action and introduce some simplification transforming to the density functional for the Fermi subsystem. Integrating over the rapid ψ_{α} -fields, changing on the scale much smaller than the average distance between the charge carriers $n^{-1/3}$, in the ladder approximation we can obtain the action for the smooth fields in the form

$$S_{eh}^{(s)} = \oint d^3 x d^3 x' \left\{ \sum_{\alpha=e,h} \bar{\psi}_{\alpha}(i\partial_t - \xi_{\alpha}) \psi_{\alpha} \delta(x - x') - \right. \\ \left. - \frac{1}{2} (\bar{\psi}_e \psi_e - \bar{\psi}_h \psi_h) \widehat{U}_C (\bar{\psi}_e \psi_e - \bar{\psi}_h \psi_h) \delta(t - t') - \frac{1}{2} \sum_{\alpha,\beta=e,h} \bar{\psi}_{\alpha} \psi_{\alpha} \Gamma_{\alpha\beta} \bar{\psi}_{\beta} \psi_{\beta} \right\} \quad (3)$$

here the fields ψ_{α} are the smooth Fermi fields. The vertex $\Gamma_{\alpha\beta}$ is the sum of the ladder diagrams. The internal Green functions of these diagrams have the large momentums, much larger than p_F , and the external lines of these diagrams have the small momentums, much smaller than the internal lines. The value of the boundary momentum Λ separating the rapid and the smooth fields obeys the inequality $\frac{\hbar}{a_B} \gg \Lambda \gg p_F$. The vertex Γ can be considered as

independent of the external momentums or frequencies due to the large values of the momentums corresponding to the internal lines of this vertex, on the assumption that this vertex does not contain the pole. The vertexes Γ_{ee} and Γ_{hh} do not contain the pole parts, but Γ_{eh} has the pole part corresponding to the formation of the bound state of the electron and the hole, i.e. the exciton,

$$\Gamma_{eh}(P, k, k') = \left(E - \frac{\vec{k}^2}{2m^*} \right) \int \frac{d^2 \vec{q}}{(2\pi)^2} \left\{ \left[\sum_n \frac{\psi_n(\vec{k}) \psi_n^*(\vec{q})}{E - E_{ex}^{(n)} + i\delta} + \int \frac{d^2 \vec{p}}{(2\pi)^2} \frac{\psi_{\vec{p}}(\vec{k}) \psi_{\vec{p}}^*(\vec{q})}{E - E_{\vec{p}} + i\delta} \right] U_C(\vec{q} - \vec{k}') \right\} \quad (4)$$

The wave functions $\psi_n(\vec{k})$ are the relative motion wave functions of the discrete part of the spectrum of the electron-hole pair, $\psi_{\vec{p}}(\vec{k})$ are the wave functions of the continuous part of the spectrum of the electron-hole pair; the energies $E_{ex}^{(n)}$ are the spectrum of the bound energies of the exciton; here we denote $E = \Omega + \mu_{e-h} - \vec{P}^2/2M$, where $\mu_{e-h} = p_F^2/2m^*$. The incoming \vec{p}_e , \vec{p}_h and outgoing \vec{p}'_e , \vec{p}'_h momentums of the ladder diagrams can be represented in the form

$$\begin{aligned} \vec{p}_e &= \frac{m_e}{M} \vec{P} + \vec{k}; & \vec{p}_h &= \frac{m_h}{M} \vec{P} - \vec{k} \\ \vec{p}'_e &= \frac{m_e}{M} \vec{P} + \vec{k}'; & \vec{p}'_h &= \frac{m_h}{M} \vec{P} - \vec{k}' \end{aligned}$$

where the momentum $\vec{P} = \vec{p}_e + \vec{p}_h$ is the exciton momentum, i.e. the total momentum of the electron-hole pair forming the bound state. The wave functions $\psi_n(\vec{r})$ and $\psi_{\vec{p}}(\vec{r})$ of the discrete and continuous parts of the internal exciton spectrum obey the Schrodinger equation

$$\begin{aligned} \left(-\frac{\nabla^2}{2m^*} - \frac{1}{|\vec{r}|} \right) \psi_n(\vec{r}) &= E_{ex}^{(n)} \psi_n(\vec{r}) \\ \left(-\frac{\nabla^2}{2m^*} - \frac{1}{|\vec{r}|} \right) \psi_{\vec{p}}(\vec{r}) &= E_{ex}(\vec{p}) \psi_{\vec{p}}(\vec{r}) \end{aligned}$$

The normalized solution of this equation in 2D for the lower energy state can be written in the momentum representation as

$$\psi_0(\vec{p}) = \frac{4\sqrt{\pi}}{(1 + \vec{p}^2)^{3/2}}$$

The normalization of this wave function has the form $\int \frac{d^2 p}{(2\pi)^2} \psi_0^2(\vec{p}) = 1$.

We suppose that the energy E is near the lower bound energy of the exciton $E_{ex}^{(0)}$. Separating the pole and the non-pole terms we can write the vertex Γ_{eh} as

$$\Gamma_{eh}(P, \vec{k}, \vec{k}') = \frac{\left(E_{ex}^{(0)} - \vec{k}^2/2m^* \right) \left(E_{ex}^{(0)} - (\vec{k}')^2/2m^* \right) \psi_0(\vec{k}) \bar{\psi}_0(\vec{k}')}{\Omega + \mu_{e-h} - \frac{\vec{P}^2}{2M} - E_{ex}^{(0)} + i\delta} + \Gamma_{eh}^{(c)} \quad (5)$$

where the term $\Gamma_{eh}^{(c)}$ is the non-pole part of the vertex Γ_{eh} , which can be considered as a constant of the order of unity $\Gamma_{eh}^{(c)} \sim 1$; the collection of the summarized frequency and the summarized momentum is $P = (\Omega, \vec{P})$. Due to the smallness of the incoming and outgoing momentums $|\vec{p}_{e,h}|, |\vec{p}'_{e,h}| \ll \hbar/a_B = 1$ the vertex Γ_{eh} can be represented in the form

$$\Gamma_{eh}(P) = \frac{F}{\Omega + \mu_{e-h} - \frac{\vec{P}^2}{2M} - E_{ex}^{(0)} + i\delta} + \Gamma_{eh}^{(c)} \quad (6)$$

where the constant F in the case of the small incoming and outgoing momentums can be represented as

$$F = \left(E_{ex}^{(0)} \psi_0(0) \right)^2$$

Near the pole of the vertex Γ_{eh} the term $\Gamma_{eh}^{(c)} \sim 1$ can be neglected. The vertex Γ_{eh} can be decoupled by the introduction of the virtual exciton field b , and the action of the system can be written in the form

$$S_{e,h,ex}^{(s)} = \oint d^3x \left\{ \sum_{\alpha=e,h} \bar{\psi}_\alpha (i\partial_t - \xi_\alpha) \psi_\alpha - \frac{1}{2} \int d^2r' (\bar{\psi}_e \psi_e - \bar{\psi}_h \psi_h) \hat{U}_C (\bar{\psi}_e \psi_e - \bar{\psi}_h \psi_h) + \right. \\ \left. + \bar{b} (i\partial_t - \xi_{ex}) b - \sqrt{F} (\bar{b} \psi_e \psi_h + \bar{\psi}_h \bar{\psi}_e b) \right\} \quad (7)$$

where

$$\xi_{ex} = \frac{\hat{p}^2}{2M} - \mu_{e-h} + E_{ex}^{(0)} \quad (8)$$

$$E_{ex}^{(0)} = -E_B$$

E_B is the bound energy of the internal exciton ground state. As it will be seen lower, the chemical potential for excitons is connected with μ_{e-h} as

$$\mu_{ex} = \mu_{e-h} - E_{ex}^{(0)} - \langle T_{e,h}^{(0)} \rangle \quad (9)$$

at that, the value $\langle T_{e,h}^{(0)} \rangle$ can be written as

$$\langle T_{e,h}^{(0)} \rangle = G_e^{(0)}(x-x') G_h^{(0)}(x-x') = \int_{|\vec{p}| > \Lambda} \frac{d^2p}{(2\pi)^2} \frac{1}{p^2/2m^*} \quad (10)$$

It is convenient to transfer from the Fermi fields of electron-hole plasma to the density variables, i.e. the density functional with respect to the Fermi fields. This transition can be accomplished by the introduction of the functional δ -function to the statistical sum

$$Z = \int D\psi_\alpha D\bar{\psi}_\alpha D b D\bar{b} \exp(iS_{e,h,ex}) = \\ = \int D\psi_\alpha D\bar{\psi}_\alpha D b D\bar{b} D n_\alpha \exp(iS_{e,h,ex}) \prod_{\alpha,x} \delta[\bar{\psi}_\alpha \psi_\alpha - n_\alpha]$$

Using the Fourier representation for the functional δ -function

$$\prod_x \delta[\bar{\psi}_\alpha \psi_\alpha - n_\alpha] = \int D V_\alpha \exp \left(-i \oint d^3x V_\alpha(x) [\bar{\psi}_\alpha \psi_\alpha - n_\alpha] \right)$$

we can rewrite the action in the form

$$S_{e,h,ex}[\psi_\alpha, b, V_\alpha, n_\alpha] = \oint d^3x \left\{ \begin{aligned} & (\bar{\psi}_e, \psi_h) \begin{pmatrix} i\partial_t - \xi_e - V_e & b \\ \bar{b} & i\partial_t - \xi_h - V_h \end{pmatrix} \begin{pmatrix} \psi_e \\ \bar{\psi}_h \end{pmatrix} + \\ & + \sum_\alpha V_\alpha n_\alpha - \frac{1}{2} (n_e - n_h) \hat{U}_C (n_e - n_h) + \bar{b} (i\partial_t - \xi_{ex}) b \end{aligned} \right\} \quad (11)$$

where

$$Z = \int D\psi_\alpha D\bar{\psi}_\alpha DbD\bar{b}Dn_\alpha DV_\alpha \exp(iS_{e,h,ex}[\psi_\alpha, b, V_\alpha, n_\alpha]) \quad (12)$$

It is convenient to introduce new variables instead of n_α and V_α , namely,

$$\begin{aligned} n_e &= \frac{n + \delta n}{2}; & n_h &= \frac{n - \delta n}{2} \\ V_e &= V + \delta V; & V_h &= V - \delta V \end{aligned} \quad (13)$$

Note that the fields δn and δV correspond to the fluctuating violation of the electro-neutrality in the electron-hole plasma. The integrals over the fields ψ_α and δn in Eq.(12) are Gauss integrals and can be calculated. As a result we obtain the action in the form

$$\begin{aligned} S_{ex}[b, V_\alpha, n_\alpha] &= -iSp \ln \left[\begin{pmatrix} i\partial_t - \xi_e - (V + \delta V) & b \\ \bar{b} & i\partial_t - \xi_h - (V - \delta V) \end{pmatrix} \right] + \\ &+ \oint d^3x \left\{ Vn + \frac{1}{2}\delta V \hat{U}_C^{-1} \delta V + \bar{b}(i\partial_t - \xi_{ex})b \right\} \end{aligned} \quad (14)$$

The supposition of the low exciton density gives the possibility to expand in powers of the field b . Integrating over the fields δV and neglecting the powers higher than b^4 we obtain

$$\begin{aligned} S_{ex}[b, V_\alpha, n_\alpha] &= -iSp \ln \left[\begin{pmatrix} i\partial_t - \xi_e - V & 0 \\ 0 & i\partial_t - \xi_h - V \end{pmatrix} \right] + \\ &+ iSp \left\{ \frac{1}{2} \left[\begin{pmatrix} 0 & b \\ \bar{b} & 0 \end{pmatrix} \hat{G}^{(0)}[V] \right]^2 \right\} + iSp \left\{ \frac{1}{4} \left[\begin{pmatrix} 0 & b \\ \bar{b} & 0 \end{pmatrix} \hat{G}^{(0)}[V] \right]^4 \right\} - \\ &- E_{corr}[V] + \oint d^3x \{ Vn + \bar{b}[i\partial_t - \xi_{ex}]b \} \end{aligned} \quad (15)$$

The functional $E_{corr}[V]$ is the correlation energy. This functional is represented as the sum of all simply connected closed diagrams dressed by the internal Coulomb interaction lines and the lines of the external field V . In the case of constant V the correlation energy $E_{corr}[V]$ can be represented in the well-known form

$$E_{corr}[V] = -i\frac{1}{2}Sp \ln [1 - \Pi[V] \hat{U}_C] = i\frac{1}{2} \int dg \int \frac{d\omega d^2k}{(2\pi)^3} \frac{U_C(k) \Pi[V]}{1 - gU_C(k) \Pi[V]} \quad (16)$$

where $\Pi[V]$ is the total polarization operator. The simple transformations of the second and third terms in (15) give the action S_{ex} in the form

$$\begin{aligned} S_{ex}[V, n] &= -iSp \ln \left[\begin{pmatrix} i\partial_t - \xi_e - V & 0 \\ 0 & i\partial_t - \xi_h - V \end{pmatrix} \right] - E_{corr}[V] + \\ &+ \oint d^3x \left\{ Vn + \bar{b}[i\partial_t - \xi_{ex}]b - \frac{1}{2}\gamma_{ex}(\bar{b}b)^2 - \gamma_{ex-e,h}(\bar{b}b)n \right\} \end{aligned} \quad (17)$$

The value $\langle T_{e,h}^{(0)} \rangle$ has the form (10). The coupling constant γ_{ex} is the exciton-exciton scattering amplitude; and $\gamma_{ex-e,h} = \gamma_{ex-e} + \gamma_{ex-h}$ where γ_{ex-e} , γ_{ex-h} are the exciton-electron and exciton-hole scattering amplitudes, respectively.

The first order expansion of the first term in Eq.(17) over the field V gives the change $Vn \rightarrow V(n - \langle n \rangle)$, where $\langle n \rangle$ is the average density of the electron-hole plasma in stationary state $\langle n \rangle = \frac{\int d^2rn(t, \vec{r})}{Vol}$, where Vol is

the volume of the system. The second order expansion of the first term in Eq.(17) in series of the field V and the integration over this field gives

$$S_{ex}[b, n] = \oint d^3x \left\{ \left[\frac{1}{2} (n - \langle n \rangle) (\Pi^{(0)})^{-1} (n - \langle n \rangle) - E_{corr}[n] \right] + \bar{b} [i\partial_t + \mu_{ex} - \mu_{e-h} - \xi_{ex}] b - \frac{1}{2} \gamma_{ex} (\bar{b}b)^2 - \gamma_{ex-e,h} (\bar{b}b) n \right\} \quad (18)$$

where $\Pi^{(0)}$ is the polarization operator of the form $\Pi^{(0)}(x-y) = -i \sum_{\alpha} G_{\alpha}^{(0)}(x-y) G_{\alpha}^{(0)}(y-x)$. Expanding in series of the field V we suppose the smallness of this field. This supposition is correct when the fluctuations of the density n are small compared with $\langle n \rangle$. This supposition is correct for the beginning of the density fluctuations growth. The derivation of the effective action in the supposition of the smooth variation of the density n at the scales larger than the average distance between particles without supposition of the smallness of the density fluctuations will be considered in the next paper.

INSTABILITY OF THE SYSTEM AND THE DENSITY MODULATION OF THE ELECTRON-HOLE PLASMA

Using the effective action for the interacting electron-hole plasma and the exciton gas $S_{ex}[b, n]$ Eq.(18) we show that the homogeneous state of this system is unstable, and due to the Coulomb correlations the low density electron-hole plasma should form the inhomogeneous state. The formation of the inhomogeneous state of the low density electron-hole plasma is the result of the tendency of the low density plasma to collapse to the liquid electron-hole drops. The interaction of this plasma with the exciton gas results in the existence of the self-consistent nonhomogeneous periodic field acting on the exciton system as the periodic external field. This periodic field results in the density modulation of the exciton system.

To analyze the effective action for simplicity we put $m = m_e = m_h = 1$. The polarization operator $\Pi^{(0)}(\omega, \vec{k})$ for $|\vec{k}| \ll p_F$ is calculated as

$$\Pi^{(0)}(\omega, \vec{k}) = \begin{cases} \frac{m}{2\pi} \frac{(kV_F)^2}{\omega^2} & \text{for } \omega \gg kV_F \\ -\frac{1}{\pi} m \left(1 + i\pi \left(\frac{\omega}{kV_F} \right) \right) & \text{for } \omega \ll kV_F \end{cases} \quad (19)$$

The correlation energy $E_{corr}[n]$ for small densities $na_B^2 \ll 1$ can be calculated as

$$E_{corr}[n] = -An^{4/3} \quad (20)$$

where the constant A is estimated as $A \sim 1$. The main contribution to the integral for $E_{corr}[n]$ (16) is given by the momentums $k_0 \sim n^{1/3} \gg p_F \sim n^{1/2}$ and the frequencies $\omega_0 \sim k_0^2$. Note that the Hartree-Fock energy for this system is estimated as

$$E_{H-F}[n] \sim -\frac{n^2}{p_F} \sim -n^{3/2} \quad (21)$$

Thus, the contribution of the Hartree-Fock energy can be neglected compared with the correlation energy $E_{H-F}(n) \ll E_{corr}(n)$ for $na_B^2 \ll 1$.

The substitution of the polarization operator $\Pi^{(0)}(\omega, \vec{k})$ Eq.(19) for $\omega \gg kV_F$ to Eq.(18) gives

$$S_{ex}[n] = \oint d^3x \left\{ \left[(\Delta n) \left[\frac{1}{2\pi} \frac{\omega^2}{(kV_F)^2} \right] (\Delta n) - E_{corr}(n) \right] + \bar{b} [i\partial_t - \xi_{ex}] b - \frac{1}{2} \gamma_{ex} (\bar{b}b)^2 - \gamma_{ex-e,h} (\bar{b}b) n \right\}$$

where $\Delta n = n - \langle n \rangle$ and expanding in Δn the functional $E_{corr}[n]$ we obtain

$$S_{ex}[\delta n] = \oint d^3x \left\{ (\Delta n) \left[\frac{1}{2\pi} \frac{\omega^2}{(kV_F)^2} - \frac{1}{2} E''_{corr}(<n>) \right] (\Delta n) + \bar{b} [i\partial_t - \xi_{ex}] b - \right. \\ \left. - \frac{1}{2} \gamma_{ex} (\bar{b}b)^2 - \gamma_{ex-e,h} (\bar{b}b) <n> - \gamma_{ex-e,h} (\bar{b}b) \Delta n \right\} \quad (22)$$

In these expressions the frequency ω should be considered as $\omega = i\partial_t$. The term linear over Δn , namely, $E'_{corr}(<n>) \Delta n$, can be omitted if the total number of particles is fixed, i.e., $\int \Delta n = 0$. The substitution of $E''_{corr}(<n>)$ (20) to (22) gives

$$S_{ex}[\delta n] = \oint d^3x \left\{ \Delta n \left[\frac{1}{2\pi} \frac{\omega^2}{(kV_F)^2} + \frac{2}{9} A <n>^{-2/3} \right] \Delta n + \bar{b} [i\partial_t - \xi_{ex}] b + \right. \\ \left. - \frac{1}{2} \gamma_{ex} (\bar{b}b)^2 - \gamma_{ex-e,h} (\bar{b}b) <n> - \gamma_{ex-e,h} (\bar{b}b) \Delta n \right\} \quad (23)$$

From the first term in Eq. (23) it can be seen that the system is unstable relative to the phonon oscillations of the electron-hole plasma density. The sound velocity of the density oscillations has the imaginary value $|c|^2 = \left(\frac{4\pi}{9} A <n>^{-2/3} V_F^2 \right)$. Note that the sound character of the considering density fluctuations is not surprising, since these fluctuations do not violate the electro-neutrality of the electron-hole plasma. This instability results in the growth of the density fluctuations of the plasma with the increment of the growth

$$\zeta = \left(\frac{4\pi}{9} A \right)^{1/2} <n>^{-1/3} kV_F \sim k <n>^{1/6} >> kV_F$$

This increment of the growth corresponds to the frequency which is much larger than kV_F , $\zeta \gg kV_F$. Due to this inequality we can use the asymptotic of the polarization operator $\Pi^{(0)}(\omega, k) \sim \frac{(kV_F)^2}{\omega^2}$ being correct for the large frequencies $\omega \gg kV_F$.

As the result, the stationary state of the electron-hole plasma should be nonhomogeneous, besides, the modulation of the density has the periodic character.

The modulated density of the electron-hole plasma results in the existence of the self-consistent field acting on the exciton system. This field has the form (23)

$$V(\vec{r}) = \gamma_{ex-e,h} \Delta n(\vec{r})$$

Note that this external for the exciton system potential plays the essential role, if the amplitude of this potential is larger than the chemical potential of the exciton gas. This condition can be obeyed for the sufficiently large density of the electron-hole plasma, i.e., for the sufficiently large pump.

At the conclusion we estimate the radius of the character non-homogeneity of the electron-hole plasma from the kinetic consideration [22]. We suppose that the kinetics of the non-homogeneity formation is analogous to the kinetics of the electron-hole liquid drop formation or the phase immiscibility and can be described by the equation

$$\partial_t (\pi R^2 n_0) = 2\pi R V_F n + 2\pi R V_{ex} n_{ex} - \pi R^2 \frac{n_0}{\tau} \quad (24)$$

The first term in the right hand side of the equation (24) is the incoming term and the second one is the outgoing term; $V_F n$ is the current of Fermi particles incoming into the drop of the radius R ; $V_{ex} n_{ex}$ is the current of excitons incoming into the drop, V_{ex} is the average velocity of excitons which can be written as $V_{ex} \sim \frac{\sqrt{\gamma_{ex} n_{ex}}}{m}$; n is the density of the electron-hole plasma in the gas phase, n_{ex} is the density of excitons, and n_0 is the density of the electron-hole liquid within the drop; τ is the lifetime of the carriers. There are no vapor terms in Eq.(24) since $T=0$, and particles can not overcome work function from the liquid drop. The equilibrium value of the radius R can be found from Eq.(24) as

$$R_0 = 2 \frac{V_F n + V_{ex} n_{ex}}{n_0} \tau \quad (25)$$

Note that the radiuses larger than R_0 are not profitable due to the different dependence on R of the incoming and outgoing terms - the first one is the surface term and the second one has the volume character. From the point of

view of the phase immiscibility the liquid phase is the electron-hole liquid drops and the gas phase is the mixture of the exciton gas and the electron-hole plasma of the small density.

Finally, the set of drops forms the periodic structure. We suppose that this is the consequence of the homogeneity of the system in average. This periodic structure forms the external periodic field for the exciton gas. If this Bose gas could be considered as the ideal, it should be localized at the minima of the periodic external potential.

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